Exercises set 02 – II.

STATISTICAL CONCEPTS FOR UNCERTAINTY ESTIMATION

Part II

Exercise 2.11

An ammeter is supposed to measure 2.00 A.
Nine measurements are carried out and the following results are obtained:

$$\bar{x} = 1.92 \text{ A}$$
$$s = 0.10 \text{ A}$$

We want to test, at a 5% significance level, whether the instrument is accurate.

1. Clearly state the null hypothesis H_0 and the alternative hypothesis H_1.
2. Calculate the test statistic t.
3. Determine the critical value of t.
4. Explain what the risk α means in this context.

Exercise 2.12

A machine must produce bars with length ≥ 100 cm.
Eight bars are measured. The statistical indicators are:

$$\bar{x} = 99.2 \text{ cm}$$
$$s = 1.5 \text{ cm}$$
$$\alpha = 5\%$$

1. State H_0 and H_1.
2. Justify the use of the Student's t distribution.
3. Calculate the test statistic t.
4. Identify the risk α.
5. Conclusion.

Exercise 2.13

Explain with a metrological example:

- Type I error
- Type II error

- How can the risk α be reduced?
- What happens to the risk β if α is reduced?

Exercise 2.14

A test leads to the rejection of H_0 with a risk of error $\alpha = 1\%$.

1. What does this 1% mean?
2. Can we affirm that H_1 is true?
3. What is the difference between significant and important?

Exercise 2.15

For each situation, indicate whether a two-tailed test or a one-tailed test should be used.

1. Check whether a measurement method gives a value different from the theoretical value.
2. Check whether a new drug increases blood pressure.
3. Check whether a machine produces less than the nominal mass.
4. Check whether an instrument is properly calibrated.

Exercise 2.16

In a physics laboratory, a quantity is measured that is assumed to have the reference value:

$$\mu_0 = 50$$

A sample of 8 independent measurements gives:

52, 49, 51, 50, 53, 48, 50, 52

It is assumed that the population follows a normal distribution.

1. Calculate the experimental mean.
2. Calculate the sample standard deviation.
3. State the hypotheses H_0 and H_1 for a two-tailed test.
4. Calculate the Student's t test statistic.
5. At the significance level $\alpha = 5\%$, decide whether the mean differs from 50.

Exercise 2.17

A machine is supposed to produce 100 g pieces.
It is suspected that the machine produces less than the nominal value.
A sample of 10 pieces is measured:

98 g, 97 g, 99 g, 100 g, 96 g, 98 g, 97 g, 99 g, 98 g, 97 g

1. Write the hypotheses of the one-tailed test.
2. Calculate the sample mean.
3. Calculate the test statistic.
4. At $\alpha = 5\%$, conclude whether the machine produces pieces less than 100 g.

Exercise 2.18

The following n values x_i are observed:

5.2, 4.9, 5.5, 5.1, 5.3, 4.8

1. Sort the data.
2. Calculate the probabilities

$$p_i = \frac{i - 0.5}{n}$$

for each rank i.

3. Determine the theoretical quantiles z_i.
4. Construct the table:

x_i	p_i	z_i

5. Explain how to draw the Henry line.
6. Conclude whether the distribution corresponds to a normal distribution.

Exercise 2.19

A student plots a Henry line for a sample. Three situations are observed:

- The points are almost aligned.
- The points form an S-shaped curve.
- The points are highly scattered.

Interpret the normality of the data in each case.

Exercise 2.20

Using the data from Exercise 2.2:

1. Calculate the frequencies $f_i$and the cumulative frequencies of each class.
2. Using the standard normal distribution table, determine the theoretical quantiles z_i.
3. Plot the corresponding Henry line.
4. What can you conclude about the normality of the data?

Problem

In a research laboratory, 15 measurements of the propagation velocity of ultrasonic waves are performed in 7 samples of the same biological tissue.
The 7 series of values (m/s) are:

Série.1	Série.2	Série.3	Série.4	Série.5	Série.6	Série.7
1539	1519	1536	1544	1529	1545	1521
1528	1533	1538	1532	1531	1532	1534
1514	1535	1532	1526	1537	1538	1544
1526	1530	1541	1538	1541	1528	1537
1524	1538	1553	1540	1529	1525	1541
1545	1528	1539	1545	1532	1538	1542
1533	1542	1531	1528	1536	1533	1535
1536	1535	1535	1534	1546	1529	1533
1533	1531	1549	1533	1541	1529	1532
1538	1541	1542	1531	1543	1524	1525
1527	1529	1526	1526	1530	1532	1532
1532	1545	1534	1544	1527	1544	1536
1533	1534	1527	1540	1535	1536	1531
1533	1532	1528	1538	1540	1518	1537
1531	1535	1538	1530	1541	1541	1523

Study of the centering of measured values:

1. What is the total number of measurements performed?
2. Calculate the experimental mean.
3. Determine the median.

Study of the dispersion of values:

1. What is the degree of freedom of all measurements?
2. Determine the minimum, the maximum, and the range.
3. Calculate the experimental variance.
4. Calculate the experimental standard deviation.
5. Calculate the coefficient of variation.

Control chart representation:

1. Calculate the Lower Warning Limit (LWL) and Upper Warning Limit (UWL).
2. Calculate the Lower Control Limit (LCL) and Upper Control Limit (UCL).
3. Plot the control chart of the measurements.
4. Conclusion.

Study of the normality of measured values:

1. Determine the optimal number of classes.
2. Group the measured values into classes.
3. Construct the grouped frequency table:

Class No.	Lower Limit	Upper Limit	Frequency	Cumulative Frequency

4. Plot the histogram of the values.
5. Plot the cumulative histogram.
6. Does the distribution appear normal?
7. Calculate the frequencies f_i and cumulative frequencies.
8. Using the standard normal distribution table, determine the theoretical quantiles z_i.
9. Group these parameters into the table:

Class No.	Lower Limit	Upper Limit	Frequency	Cumulative Frequency	z_i

10. Plot the corresponding Henry line.
11. Give the equation of the line and its coefficient of determination.
12. Does this line allow you to conclude that the data are normally distributed?
13. From the graph, deduce the mean and standard deviation.

Study of each measurement series:

1. Calculate the mean value and standard deviation of each measurement series.
2. Bound the results obtained for each series with a 95.45% confidence level, justifying the probability law used.
   Results should be written in the form:
   $$V_i = \bar{v}_i \pm U(v_i)$$
   Specify the expression of $U(v_i)$.
3. Summarize the results in the table:

	Serie.1	Serie.2	Serie.3	Serie.4	Serie.5	Serie.6	Serie.7
$\bar{v}_i$ (m/s)							
s_i (m/s)							
V_i (m/s)							

4. Bound the result concerning all values together with the same confidence level in the form:
   $$V = \bar{v} \pm U(v)$$
   Specify the expression of $U(v)$and the probability distribution used.